CIVE70095: Structural Dynamics

Lecture 1: Dynamic loads and responses

1. Dynamic loads
2. Fourier Transform
3. Discrete-time Fourier Transform

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Key concepts

- Different types of dynamic loads (responses)
- Functions (loads or responses) of arbitrary complexity may be modelled as linear sums of sines and cosines.
- Fourier Transform and why we use it.
- Discrete-time Fourier Transform

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CIVE70095: Structural Dynamics

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1. <u>Dynamic loads</u>
2. Fourier Transform
3. Discrete-time Fourier Transform

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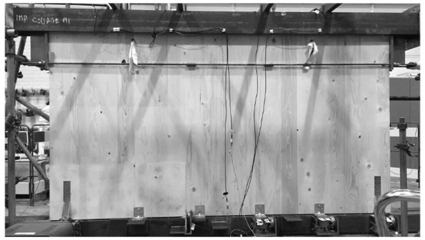
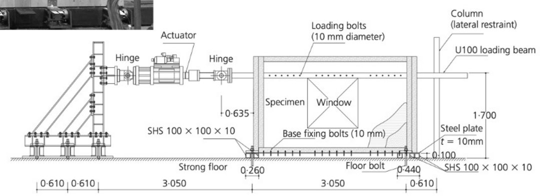
Large-scale tests on CLT panels





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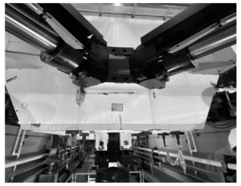
Full-scale tests on a tall timber building



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Dynamic loads - What is a dynamic load?

- Static loads → vectors (amplitude, position, direction)
- Dynamic loads → **temporal** nature:

  amplitude, position, direction (a combination or all) **change in time**

… at a rate sufficient to cause Inertial Forces

… those inertial forces also change in time

… the stresses and strains caused also change in time

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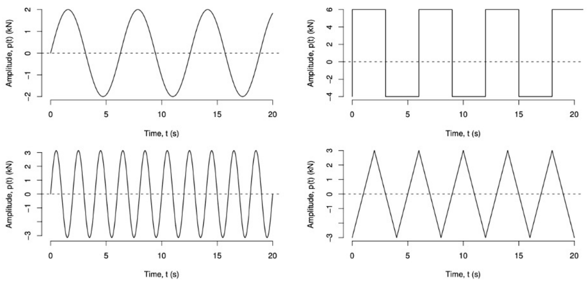
Types of dynamic loads: i) Periodic



(a) Harmonic loads (b) Non-harmonic loads

Figure 1.1: Periodic loads

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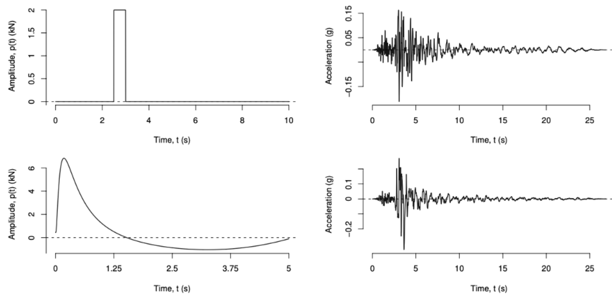
Types of dynamic loads: ii) Aperiodic



(a) Impulsive loads (b) Transient (seismic) loads

Figure 1.2: Aperiodic loads

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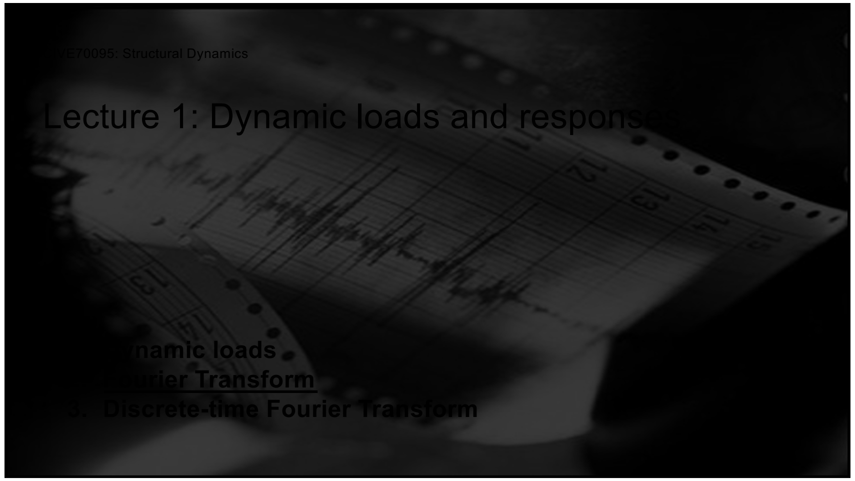

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The time-frequency duality: a matter of perspective

- Time ←→ frequency
- 2 ways of looking at a structural response/load that are interchangeable
- No information is lost in changing from one to the other (in principle).

- The advantage is that changing the perspective the solution to a dynamic problem can become simpler and/or clearer (we will see this in our last lecture)

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The time domain

- The traditional way of observing phenomena (loads, responses)
- Is a record of what happens to a given parameter versus time
- Issues with measuring it:
  - Period : long enough
  - Amplitude : large enough
  - Force : strong enough
  - Indirect measurement : signals

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The frequency domain

- By adding up sine waves we can generate any waveform that exists in the real world
- By picking the amplitudes, frequencies and phases correctly we can (in theory) reproduce it identically
- The same applies the other way round: a real world signal (load or response history) can be broken down into a series of sine waves

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The frequency domain

https://bit.ly/3YnWX6d

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Representation of dynamic loads as harmonics

- Principle of superposition – **linear structures**

… this idea can be extended to dynamic problems!

- Decompose the load into a trigonometric series
- Find the response to each individual harmonic
- Sum all the responses to obtain the total response

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Representation of dynamic loads as harmonics

$$g(t) = A \sin(2\pi\nu t + \phi)$$

- A is the amplitude
- ν is the frequency measured in cycles per second
- ϕ is the phase (responsible for getting values other than 0 at $t = 0$)
- $\omega = 2\pi\nu$ is the circular frequency measured in radians per second
- In addition, we can define period as $T = 1/\nu$, the inverse of the frequency

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Representation of dynamic loads as harmonics

- Any periodic function (load) can be represented as a linear combination of sines and cosines

$$f(t) = A_0 + \sum_{k=1}^{n} (A_k \cos(2\pi\nu_k t) + B_k \sin(2\pi\nu_k t))$$

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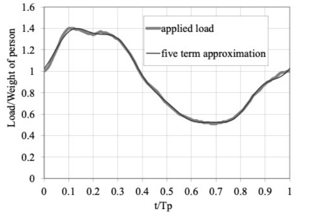
Representation of dynamic loads as harmonics

The case of a periodic force from a person walking



$$f(t) = A_0 + \sum_{k=1}^{n} (A_k \cos(2\pi\nu_k t) + B_k \sin(2\pi\nu_k t))$$

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Representation of dynamic loads as harmonics

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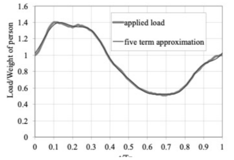
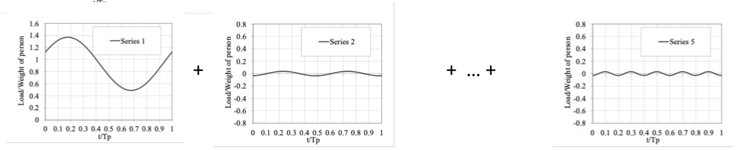


$$f(t) = A_0 + \sum_{k=1}^{n} (A_k \cos(2\pi\nu_k t) + B_k \sin(2\pi\nu_k t))$$



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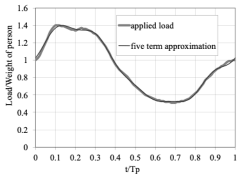
Representation of dynamic loads as harmonics

The case of a periodic force from a person walking



$$f(t) = A_0 + \sum_{k=1}^{n} (A_k \cos(2\pi\nu_k t) + B_k \sin(2\pi\nu_k t))$$

$$f_1(t) = 0.929 + 0.1943\cos(2\pi t/T_p) + 0.3954\sin(2\pi t/T_p) - 0.0352\cos(4\pi t/T_p) + 0.007\sin(4\pi t/T_p)$$
$$-0.0065\cos(6\pi t/T_p) - 0.0073\sin(6\pi t/T_p) - 0.0256\cos(8\pi t/T_p) + 0.0141\sin(8\pi t/T_p)$$
$$-0.0304\cos(10\pi t/T_p) + 0.0028\sin(10\pi t/T_p)$$

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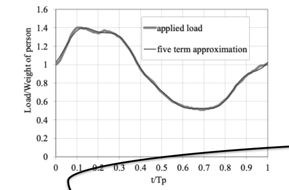
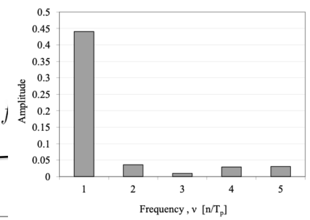
Representation of dynamic loads as harmonics

The case of a periodic force from a person walking





$+ B_k \sin(2\pi\nu_k t))$

k	Frequency (ν_k)	Cosine Amplitude (A_k)	Sine Amplitude (B_k)	Amplitude of the harmonic ($\sqrt{A_k^2 + B_k^2}$)
1	$1/T_p$	0.1943	0.3954	0.4406
2	$2/T_p$	-0.0352	0.007	0.0359
3	$3/T_p$	-0.0065	-0.0073	0.0098
4	$4/T_p$	-0.0256	0.0141	0.0292
5	$5/T_p$	-0.0304	0.0028	0.0305

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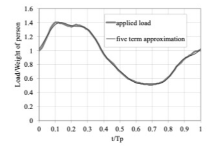
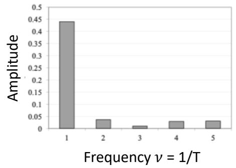
The Fourier Transform

- Periodic → Fourier expansion





- Aperiodic → Fourier Transform

The **Fourier Transform** is a mathematical tool that **takes a function in time**, measures every harmonic component, **and returns** the information in terms of **frequencies**.

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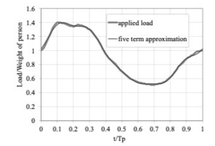
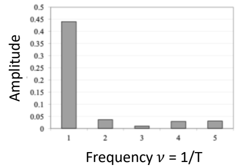
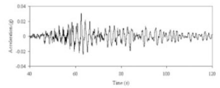
The Fourier Transform

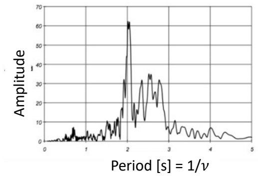
- Periodic → Fourier expansion





- Aperiodic → Fourier Transform





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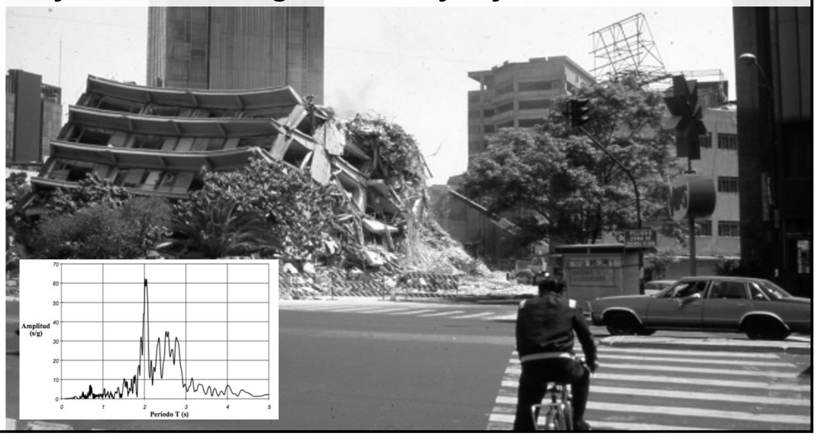
Why we need the ingredients anyway?



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Principle of superposition: Linear structures

… this idea can be extended to dynamic problems!

- Decompose the load into a trigonometric series
- Find the response to each individual harmonic
- Sum all the responses to obtain the total response

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The Fourier Transform

The **Fourier Transform** of a function in the time domain $f(t)$ is defined as:

$$F(\nu) = \int_{-\infty}^{\infty} f(t) e^{-2\pi i \nu t} dt$$

while the **Inverse Fourier Transform** of a function in the frequency domain $F(\nu)$, is:

$$f(t) = \int_{-\infty}^{\infty} F(\nu) e^{2\pi i \nu t} d\nu$$

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The mathematics of the Fourier Transform

$$e^{i\theta} = cos\theta + isin\theta \qquad e^{-i\theta} = cos\theta - isin\theta$$

$$cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2} \qquad sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

$$f(t) = A_0 + \sum_{k=1}^{n} (A_k \cos(2\pi\nu_k t) + B_k \sin(2\pi\nu_k t))$$

$$f(t) = A_0 + \sum_{k=-n}^{n} \left[C_k e^{2\pi i \nu_k t}\right]$$

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The mathematics of the Fourier Transform

- Formulation as a sum of exponentials

$$f(t) = A_0 + \sum_{k=1}^{n} (A_k \cos(2\pi\nu_k t) + B_k \sin(2\pi\nu_k t))$$

$$f(t) = A_0 + \sum_{k=-n}^{n} \left[C_k e^{2\pi i \nu_k t}\right]$$

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3	$3/T_p$	-0.0065	-0.0073
4	$4/T_p$	-0.0256	0.0141
5	$5/T_p$	-0.0304	0.0028

$$\nu_k = -\nu_{-k} \quad ; \quad k < 0$$

k	Frequency (ν_k)	C_k
-5	$-5/T_p$	-0.0152 +i 0.0014
-4	$-4/T_p$	-0.0128 +i0.00705
-3	$-3/T_p$	-0.00325 -i0.00365
-2	$-2/T_p$	-0.0176 +i0.0035
-1	$-1/T_p$	0.09715 +i0.1977
0	0	0
1	$1/T_p$	0.09715 -i0.1977
2	$2/T_p$	-0.0176 -i0.0035
3	$3/T_p$	-0.00325 +i0.00365
4	$4/T_p$	-0.0128 -i0.00705
5	$5/T_p$	-0.0152 -i0.0014

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The mathematics of the Fourier Transform

- Formulation as a sum of exponentials

$$f(t) = A_0 + \sum_{k=-n}^{n} \left[C_k e^{2\pi i \nu_k t}\right]$$

Polar notation: amplitude, vector

$$x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

$$r = |x + iy| = \sqrt{x^2 + y^2} \quad \text{and} \quad \tan\theta = \frac{y}{x}$$

$$C_k e^{2\pi i \nu_k t} = r_k e^{i\phi_k} e^{2\pi i \nu_k t} = r_k e^{i(2\pi\nu_k t + \phi_k)}$$

$$f(t) = A_0 + \sum_{k=-n}^{n} r_k e^{i(2\pi\nu_k t + \phi_k)}$$

C_k is a complex exponential

r_k is modulus

$$f(t) = \int_{-\infty}^{\infty} F(\nu) e^{2\pi i \nu t} d\nu$$

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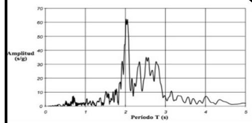
The Fourier Transform

$$f(t) = A_0 + \sum_{k=1}^{n} (A_k \cos(2\pi\nu_k t) + B_k \sin(2\pi\nu_k t))$$

$$F(\nu) = \int_{-\infty}^{\infty} f(t) e^{-2\pi i \nu t} dt$$



$$f(t) = \int_{-\infty}^{\infty} F(\nu) e^{2\pi i \nu t} d\nu$$

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The Fourier Transform

- **Properties**

Property	$f(t)$	$F(\nu)$
1. Linearity	$af_1(t) + bf_2(t)$	$aF_1(\nu) + bF_2(\nu)$
2. Convolution theorem	$f_1(t) * f_2(t)$	$F_1(\nu)F_2(\nu)$
3. Product theorem	$f_1(t)f_2(t)$	$F_1(\nu) * F_2(\nu)$
4. Time shifting	$f(t - t_0)$	$F(\nu)e^{-2\pi i \nu t_0}$
5. Frequency shifting	$f(t)e^{-2\pi i \nu_0 t}$	$F(\nu - \nu_0)$
6. Scaling	$f(at)$	$\|a\|^{-1}F(\nu/a)$

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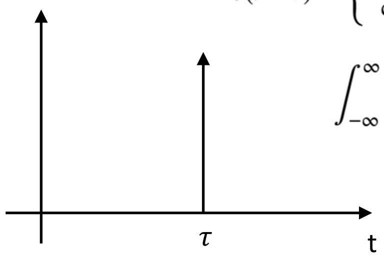
Example: FT of the Dirac Delta Function

- The Dirac Delta function

$$\delta(t-\tau) = \begin{cases} 0 & \text{for } t \neq \tau \\ \infty & \text{for } t = \tau \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t-\tau) dt = 1$$



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Example: FT of the Dirac Delta Function

- The Dirac Delta function

$$\delta(t-\tau) = \begin{cases} 0 & \text{for } t \neq \tau \\ \infty & \text{for } t = \tau \end{cases}$$

It is called a function but we can think of it as an operator, and one that has a very important characteristic:

$$\int_{-\infty}^{\infty} \delta(t-\tau) f(t) dt = f(\tau)$$

So the Dirac Delta can be used to 'pick out' individual values of a function

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Example: FT of the Dirac Delta Function

- **FT** of $\delta(t-a)$ $\qquad F(\nu) = \int_{-\infty}^{\infty} \delta(t-a) e^{-2\pi i \nu t} dt$

$$\int_{-\infty}^{\infty} \delta(t-\tau) f(t) dt = f(\tau) \longrightarrow F(\nu) = \int_{-\infty}^{\infty} \delta(t-a) e^{-2\pi i \nu t} dt = \boxed{e^{-2\pi i \nu a}}$$

- **IFT** of $e^{-2\pi i \nu a}$:

$$f(t) = \int_{-\infty}^{\infty} F(\nu) e^{2\pi i \nu t} d\nu = \int_{-\infty}^{\infty} e^{-2\pi i \nu a} e^{2\pi i \nu t} d\nu = \boxed{\delta(t-a)}$$

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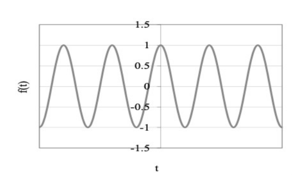
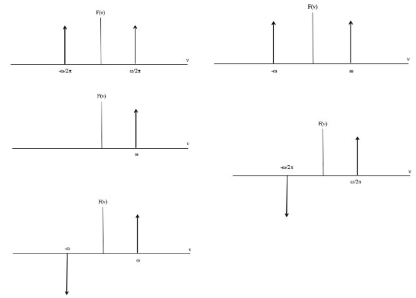
Which one of these plots corresponds to the Fourier spectra of the cosine function?

$f(t) = \cos(2\pi a t)$





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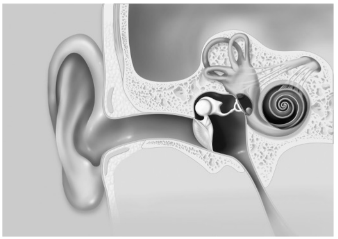
The frequency domain: a familiar domain



- Cochlea: tapered membrane coil
- Real time Fourier decomposition
- Stethoscope
- Frequency examples

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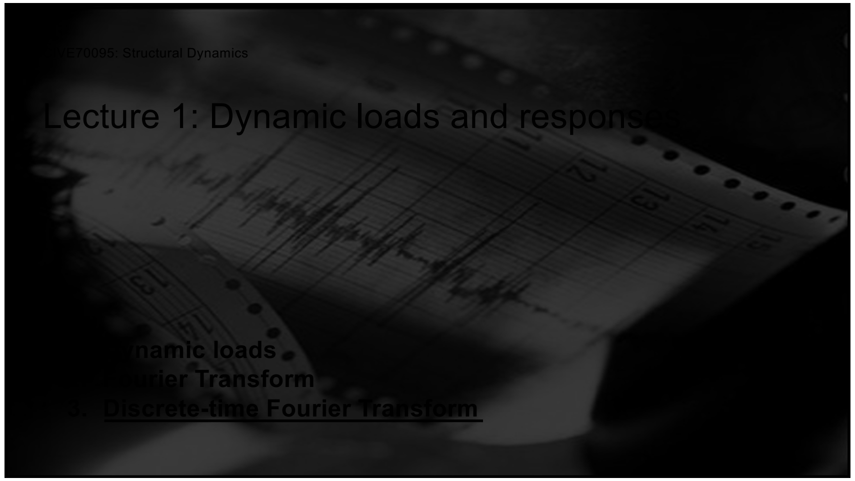

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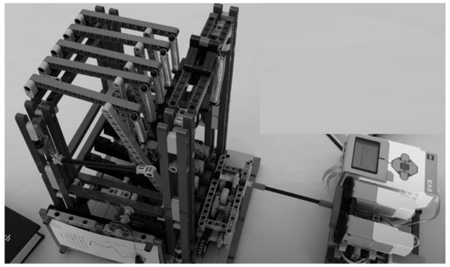
Fourier analysis is time-consuming

Kelvin's harmonic analyser (1878)



Someone's (probably a geek) Lego Mindstorms version (2022)

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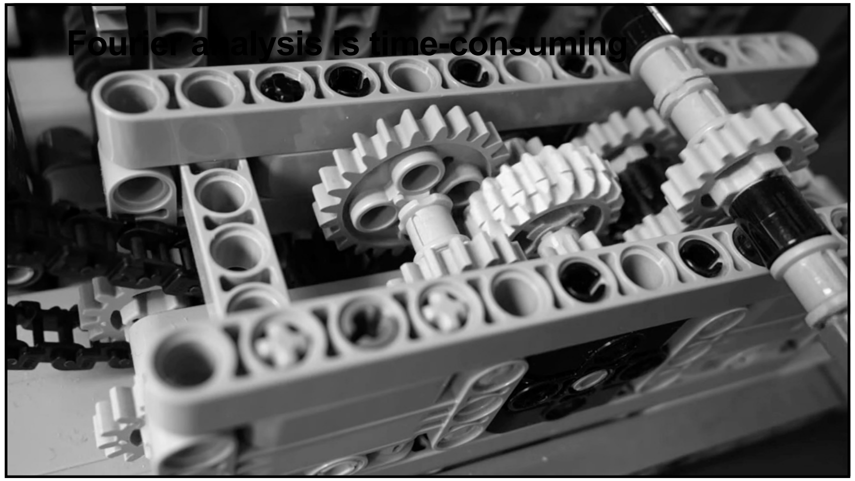
Fourier analysis is time-consuming



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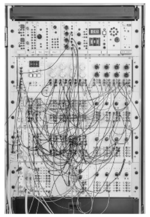
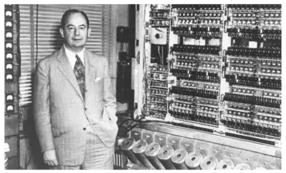
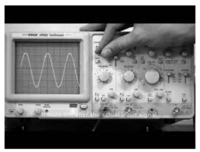
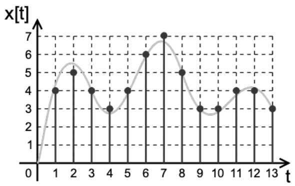
Continuous to discrete: analogue to digital









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The Discrete-time Fourier Transform

- The DTFT:

$$X(e^{i\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-i\omega n}$$

- The synthesis equation:

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{i\omega})e^{i\omega n}d\omega$$

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The Discrete-time Fourier Transform

- The main differences between discrete-time and continuous -time FT:
  - Periodicity of the DFT
  - Finite interval of integration of the synthesis equation

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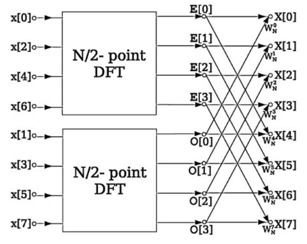
Fast Fourier Transform (FFT) Algorithm

James W. Cooley, IBM

John Tukey, AT&T Bell Labs



https://www.youtube.com/watch?v=iTMn0Kt18tg.

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Tutorials

Knowing <> understanding

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