Structural Dynamics

Tutorial 1 - Dynamic loads and responses

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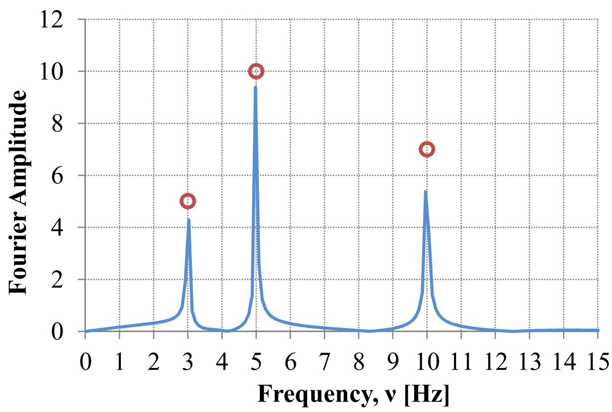
1. Given the load function: $f(t) = 0.5\sin(\pi t) + 2\sin(4\pi t) + 4\cos(2\pi t)$. Express the frequency contents of $f(t)$ in terms of a sum of exponentials.

2. Obtain the Fourier Transform of a Pulse function defined as:

$$p_T(t) = \begin{cases} A & |t| \leq T/2 \\ 0 & |t| > T/2 \end{cases}$$

   And plot both $p_T(t)$ and $P_T(\nu)$ for ($A = 1$kN, $T = 2$s) and ($A = 4$kN, $T = 0.5$s). What are the implications of these plots?

3. Consider the Fourier Amplitude spectrum shown below. If one were to only consider the frequencies where the three large peaks occur, what set of phase angles would result in the maximum possible value of the load for this particular spectrum? What would this maximum load amplitude be?



   Note that the open circles represent the theoretically exact values for this particular signal, while the solid line represents an N-point representation of this signal (this is why the peaks do not perfectly reach these theoretical points). You can assume $\phi_1 = 0$.

4. Find the Fourier Transform $X(i\omega)$ of: $x(t) = e^{-at}u(t)$ for $a > 0$, where $u(t)$ is the unit-step function defined as: $u(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$
   And plot its magnitude and phase.

5. Find and plot the DFT of $x[n] = a^{|n|}$, with $|a| < 1$ for $0 < a < 1$.